More Recursion

2024 Winter APS 105: Computer Fundamentals
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Lecture 26
1.0.0

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A Recursive Function Calls Itself

We need two things:

1. a base case: a simple solution we know
2. a recursive step: reduces the problem to a smaller version of itself

Computing the Greatest Common Divisor (GCD)

The GCD of two integers `a` and `b`, is the largest integer `d` that is a divisor of both `a` and `b`

We'll assume all integers are positive and greater than 0

The Euclidean Algorithm for Finding the GCD

Find the largest common divisor, `d`, of integers `a` and `b`

Given: `a` $\geq$ `b`

Replace `gcd(a, b)` with `gcd(b, a % b)`

until `gcd(d, 0)`, where `d` is the GCD

We can write a recursive solution to this problem!

Finding the GCD in C

```
int gcd(int a, int b) {
    if (b == 0) {
        return a;
    }
    if (a >= b) {
        return gcd(b, a % b);
    }
    else {
        return gcd(b, a);
    }
}
```

For more practice, you could try to solve this using a loop instead

Can We Count to 5 Recursively?

Think about how we'd write this function

Counting from 1 to 5 Recursively

```c
#include <stdio.h>
#include <stdlib.h>

void count(int n) {
    if (n <= 0) {
        return;
    }
    printf("%d\n", n);
    count(n - 1);
}

int main(void) {
    count(5);
    return EXIT_SUCCESS;
}
```

What happens if we move `printf` to AFTER the recursive call?

Moving `printf` Counts from 5 to 1 Instead

```c
#include <stdio.h>
#include <stdlib.h>

void count(int n) {
    if (n <= 0) {
        return;
    }
    count(n - 1);
    printf("%d\n", n);
}

int main(void) {
    count(5);
    return EXIT_SUCCESS;
}
```

What About Computing the Sum of an Array Recursively?

We can use our same two rules for this as well!

Computing the Sum of An Array, Recursively

```
int sum(int *array, int arrayLength) {
    if (arrayLength == 0) {
        return 0;
    }
    else {
        return array[0] + sum(array + 1, arrayLength - 1);
    }
}
```

Maybe We Think of Another Solution

```
int sum(int *array, int arrayLength, int currentSum) {
    if (arrayLength == 0) {
        return currentSum;
    }
    else {
        return sum(array + 1, arrayLength - 1, array[0] + currentSum);
    }
}
```

Having an Extra Argument Can Be Confusing

Instead of:

```
int sum(int *array, int arrayLength, int currentSum);
```

It would be easier to use:

```
int sum(int *array, int arrayLength);
```

We can create a “helper” function that has all the arguments, and use it in our easier to use function

Another Sum Solution with a Helper Function

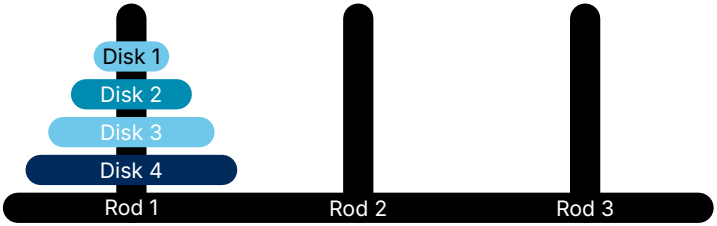
```
int sum_helper(int *array, int arrayLength, int currentSum) {
    if (arrayLength == 0) {
        return currentSum;
    }
    else {
        return sum_helper(array + 1, arrayLength - 1, array[0] + currentSum);
    }
}

int sum(int *array, int arrayLength) {
    return sum_helper(array, arrayLength, 0);
}
```

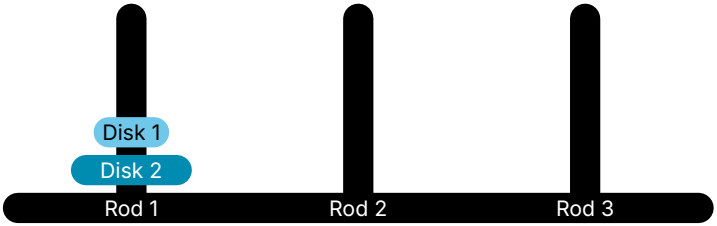
Can We Solve the Tower of Hanoi Recursively?

You want to move the tower of disks from rod 1 to 3 (peg numbers in white)

1. You can only move one disk at a time
2. You can move the top disk from a rod and place it at the top of another rod
3. You cannot place a larger disk on top of a smaller one



Let’s Solve the Tower of Hanoi with 2 Disks



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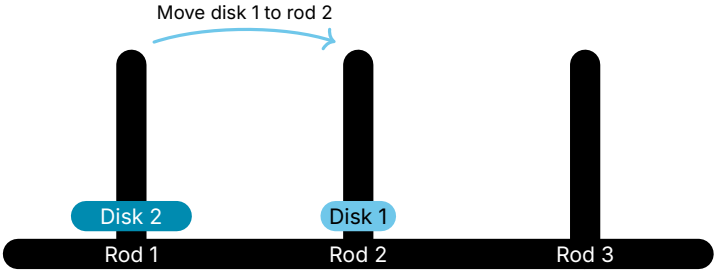

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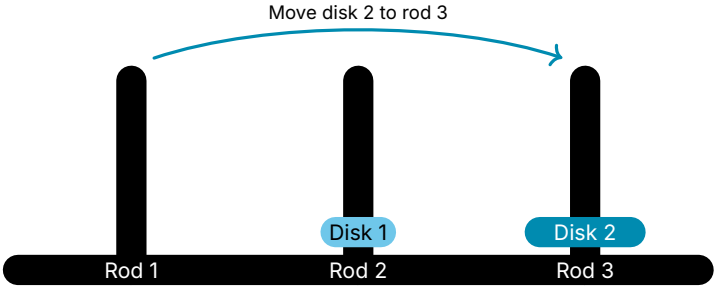

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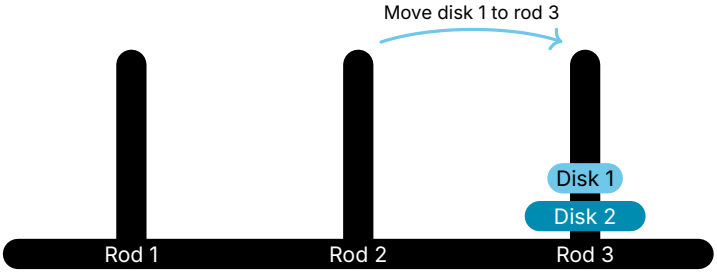
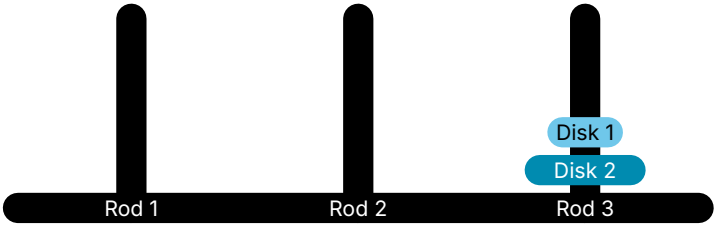
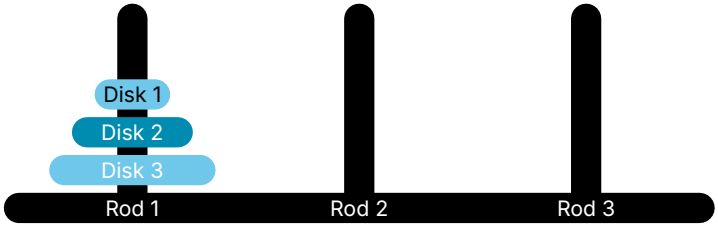


Let’s Solve the Tower of Hanoi with 2 Disks

What happens if we move disk 1 to rod 3 first?



Let’s Solve the Tower of Hanoi with 3 Disks



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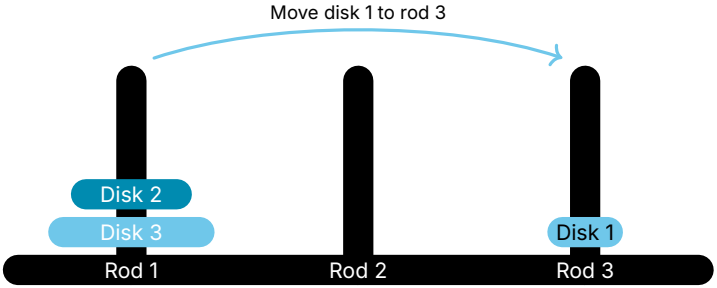

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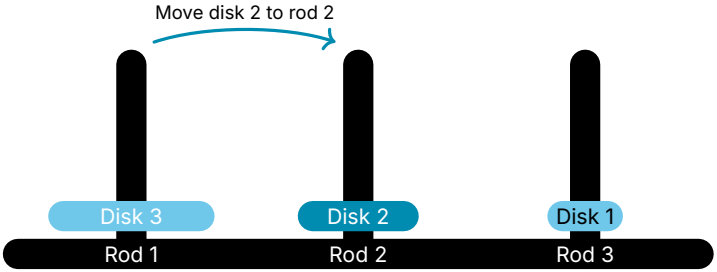

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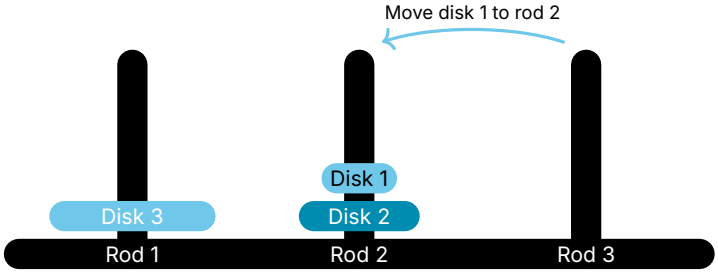

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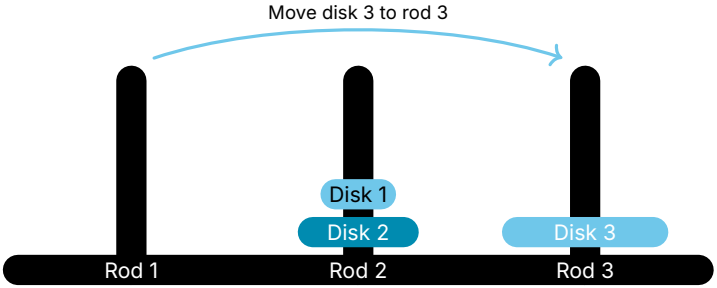

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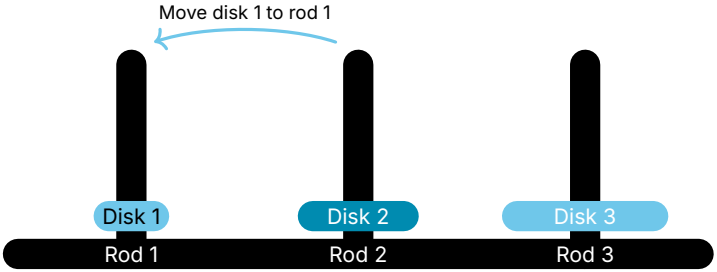

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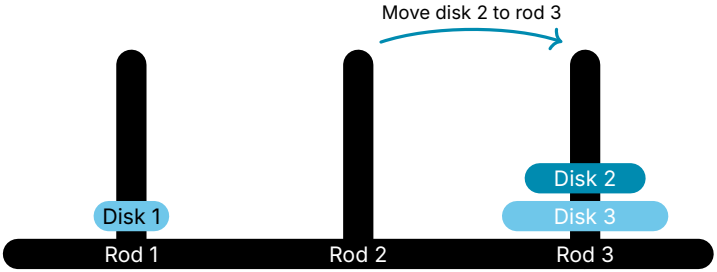

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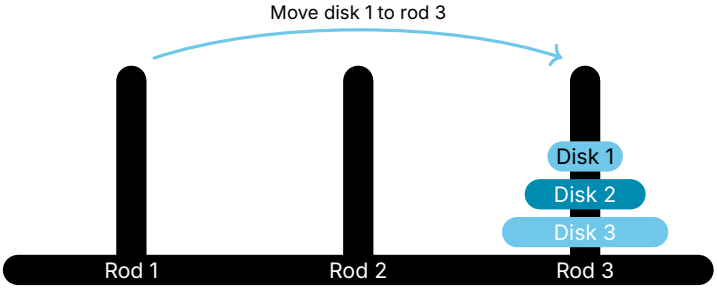
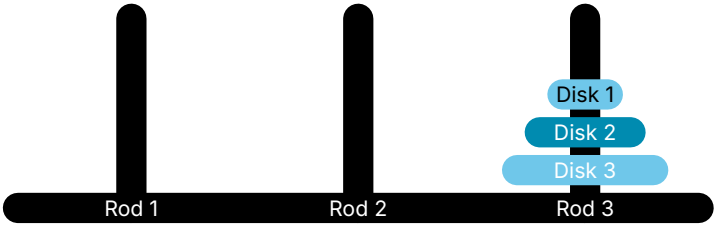


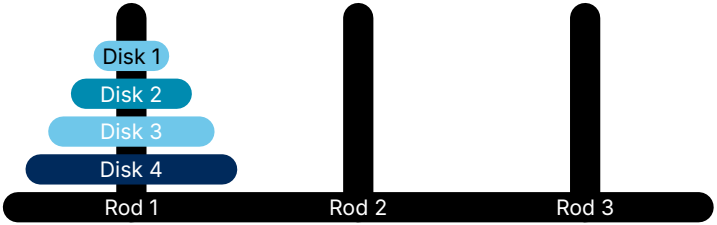
Let’s Solve the Tower of Hanoi with 3 Disks

We can solve this in 7 steps. Did you notice a recursive pattern?

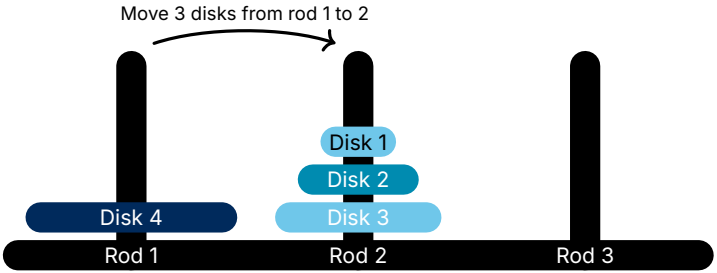


Discovering the Recursive Pattern to the Tower of Hanoi

Problem: Move 4 disks from rod 1 to 3



Discovering the Recursive Pattern to the Tower of Hanoi



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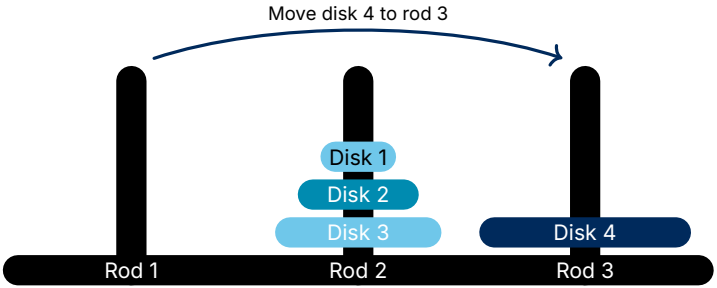
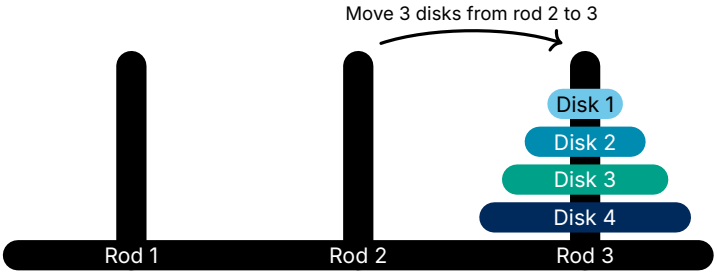


Discovering the Recursive Pattern to the Tower of Hanoi



We Can Solve the Tower of Hanoi with 2 Subproblems

Generalize the rods: from, to, and spare

We have a subproblem:
any smaller disk can go on top of the one we're solving for

To move n disks from **from** to **to** using **spare** as spare

1. Move n − 1 disks from **from** to **spare** using **to** as spare
2. Move disk n from **from** to **to**
3. Move n − 1 disks from **spare** to **to** using **from** as spare

What are we missing?

Our Tower of Hanoi Solution is Compact

```
int hanoi(int disks, int from_rod, int to_rod, int spare_rod) {
    /* Base case */
    if (disks == 0) {
        return 0;
    }
    /* Recursive steps */
    int steps = hanoi(disks - 1, from_rod, spare_rod, to_rod);
    printf("Move disk %d to rod %d\n", disks, to_rod);
    steps += 1;
    steps += hanoi(disks - 1, spare_rod, to_rod, from_rod);
    return steps;
}
```